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Estimated model parameters of incomplete gamma and inverse polynomial functions and comparison of the model's lactation yield prediction potential in zebu and crossbred cattle

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Abstract

The study was conducted to estimate model parameters of Incomplete Gamma (IG) (b=1) and Inverse Polynomial (IP) functions of a lactation curve and the models lactation yield prediction potential based on milk data from Bako Agricultural Research centre and Debre Zeit Research Station. The models were fitted to weekly total milk data of indigenous and crossbred cows. Five model parameters ("a", "c", "A_0", "A_1" and "A_2") of the two models, estimated from regression of weekly total milk data on weekly interval from calving, were analysed for the effects of genetic and non-genetic factors. Based on the estimated model parameters for each cow, lactation yield was predicted and the deviation of the predicted from the actual yield estimated. The predicted lactation yield and percent deviation of the predicted from the actual lactation milk yield were analysed using the General Linear Model. The overall least square mean values of "a", "c", "A_0", "A_1" and "A_2" were 2.559 ± 0.01, 0.103 ± 0.001, 0.379 ± 0.024, -0.076 ± 0.007 and 0.006 ± 0.001, respectively. All parameters were significantly ($p < 0.001$) affected by sire breed and calving year. Besides, calving season ($p < 0.001$) and parity of the cow (at least $p < 0.05$) significantly affected "a" and "c" only. Crossbred cows had significantly ($p < 0.001$) higher "a" and "A_1" and lower "c", "A_0" and "A_2" values than the indigenous breeds. Horro, as a sire breed, had significantly higher "a", "A_0" and "A_2" and lower "A_1" than Boran cows but not in the value of the parameter "c". Crossbred cows, however, were not significantly different among each other in the values of all parameters. Cows that calved in 1989 had significantly ($p < 0.001$) the highest "a"(2.750 ± 0.097), "A_2" ($0.019 \pm$

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0.004) and "c" (0.132 ± 0.009), while the highest values of "A_0" (1.01 ± 0.11) and the lowest "A_1"(-0.238 ± 0.03) were recorded for cows that calved in 1987. Significantly ($p < 0.001$) highest values of "a" (2.543 ± 0.023) and "c" (0.115 ± 0.002) were obtained for cows that calved during Arfasa (short rainy season). The lowest values of "a" (2.156 ± 0.031) and "c" (0.102 ± 0.003) were recorded for cows in the first parity ($p < 0.001$). The parameter "c" showed a linear trend with an increase in parity order. The overall mean actual lactation milk yield, and lactation milk yield predicted from IG (b = 1) and IP are 1522.3 ± 17.18, 1540.3 ± 15.91 and 1360.9 ± 110.9 kg, respectively. The IG (b = 1) over-predicted the overall least square mean lactation milk yields by about 2.2 ± 2.33 % while the IP under-predicted least square mean lactation milk yield by 15.5 ± 9.61 %. The IG (b = 1) over-predicted least square mean lactation yields by 1.1 to 5.0% for all sire breeds, dam breeds, locations and parities, while the IP under-predicted least square mean lactation yield of all sire breeds except Horro and that of both dam breeds and locations. The significantly ($p < 0.001$) highest deviation was recorded for Horro cows (5.0 ± 0.69 %) compared to the other sire breeds. Comparison of dam breeds, locations and parities indicated that the percent deviation was significantly ($p < 0.05$) higher for Horro than Boran cows (3.4 vs 1.1%); for Bako than Debre Zeit herd (3.1 vs 1.5 %) and for cows in later than earlier parities. In the case of the IP, significantly ($p < 0.05$) highest deviation was observed for Boran cows (190.9 ± 50.9 %) compared to the other sire breeds. From this study it can be concluded that the IG (b = 1) better fitted to the weekly total milk data and enabled prediction of lactation milk yield with minimum biased compared to the IP and could be recommended for fitting to the lactation data of indigenous and crossbred cows of the study areas.

Keywords/phrases: Lactation curve, model, incomplete gamma, inverse polynomial

Introduction

A mathematical model of the lactation curve provides summary information on milk production, which is useful in making management and breeding decisions. Different studies (Wood, 1969; Madalena *et al.*, 1979; Rowlands *et al.*, 1982; Papajcsik and Bodero, 1988; Sherchand *et al.*, 1995; Perochon *et al.*, 1996; Olori *et al.*, 1999) have been conducted on lactation curves, and mathematical functions have been derived to describe these lactation curves. The usefulness of any mathematical model depends on how well it can mimic the biological process of milk production and adjust for factors affecting it. The objective of modelling lactation curves is to fit a mathematical function that can predict the yield on each day of lactation with minimum error. This can be used as a basis for decisions to cull or retain breeding stock (Papajcsik and Bodero, 1988; Sherchand *et al.*, 1995). Besides, the shape of the curve indicates to the dairy producer the needed changes in feeding management.

In a previous work done to compare the fitness of five models to lactation data of indigenous and crossbred cows, Gebregziabher *et al*. (2003b) selected the IG (b = 1) ($Y_N = AN^b\exp(-cN)$) and IP ($Y_N = N(A_0 + A_1N + A_2N^2)^{-1}$) for their better fitness and practical applicability for data from both indigenous and crossbred cows. These models have a total of five parameters ("a", "c", "A_0", "A_1" and "A_2"). The parameter "ln(A)" or "a" in the IG (b = 1) is defined as "the scale of production" (Wood, 1972) or "initial milk production" (Madalena *et al.*, 1979) while "c" represents the rate of change in the declining phase of lactation curve (Yadav *et al.*, 1977). The parameter "a" is a scaling factor which is responsible for lowering or raising the lactation curves and will not change the general shape of the lactation curve (Batra, 1986), but "c" influences the shape of the lactation curve (Yadav *et al.*, 1977). In the IP model the parameters measure the rate of increase to peak production ("A_0"), the average slope of the lactation curve ("A_1") and the rate of decline after the peak yield is attained ("A_2"). In this study model parameters of the selected functions, which enable to construct lactation curve equations, are estimated and analysed for the effects of genetic and non-genetic factors on these parameters. Besides, the models power to predict lactation yield was compared.

Materials and Methods

The study was conducted based on existing data from Bako Agricultural Research Centre of Oromia Agricultural Research Institute and Debre Zeit Research Station of the International Livestock Research Institute (ILRI). Details of the two centres climatic condition, livestock management, breeding and health care are reported in previous works (Gemechu, 1992; Gebregziabher and Mulugeta, 1996). Lactation data of pure Horro and Boran and their F_1 crosses with Jersey, Friesian and Simmental exotic sire breeds were used for the study. The two models used for this study are IG (b = 1) with two parameters ($Y_N = AN^b\exp(-cN)$ b = 1; Papajicsik and Bodero, 1988) and IP ($Y_N = N(A_0 + A_1N + A_2N^2)^{-1}$; Nelder, 1966). In the models "Y_N" represents weekly total milk yield recorded at week "N" interval from calving. The models were fitted to weekly total milk data using the regression procedure of the Statistical Analysis System (SAS, 1999). The IG (b = 1) was transformed to its linear form using a natural logarithmic transformation before fitting to the data, while the IP was not transformed. Five model parameters, two ("ln(A)" or "a" and "c") from the IG (b = 1) and three ("A_0", "A_1" and "A_2") from IP functions were estimated and analysed for different fixed effect using the General Linear Model (GLM) of the Statistical Analysis System (SAS, 1999). The GLM included fixed effects of sire and dam breed, location, calving season and year. Detailed descriptions of these fixed effects are presented in Gebregziabher *et al.*(2003a). Using the estimated model parameters obtained from each lactation, lactation milk yield was predicted for each cow. Then the predicted and actual lactation yields, lactation length and percentage deviation of predicted from the actual lactation milk yield were calculated and analysed for the effects of different factors.

Results

The overall least square mean values of "a", "c", "A_0", "A_1" and "A_2" were 2.559 ± 0.01, 0.103 ± 0.001, 0.379 ± 0.024, -0.076 ± 0.007 and 0.006 ± 0.001, respectively (Table 3). All parameters ("a", "c", "A_0", "A_1" and "A_2") were significantly ($p < 0.001$) affected by sire breed and calving year (Table 1) while the effect of calving season ($p < 0.001$) and parity of the cow ($p < 0.05$) were significant only for the parameters "a" and "c". Crossbred cows had significantly ($p < 0.001$) higher "a" and "A_1" and lower "c", "A_0" and "A_2" values than indigenous breeds. Horro, as a sire breed, had significantly ($p < 0.001$) higher "a", "A_0" and "A_2" and lower "A_1" than Boran cows but not in the value of the parameter "c". Crossbred cows were not significantly different among each other in the values of all parameters. The lowest values of "a" (2.156 ± 0.031) and "c" (0.102 ± 0.003) were recorded for cows in the first parity ($p < 0.001$). The parameter "c" showed a linear trend with an increase in parity order. Significantly ($p < 0.001$) highest "a" (2.543 ± 0.023) and "c" (0.115 ± 0.002) were observed for cows that calved during Arfasa (short rain season). Cows that calved in 1989 had significantly ($p < 0.001$) highest "a"(2.750 ± 0.097), "A_2" (0.019 ± 0.004) and "c" (0.132 ± 0.009), while the highest values of "A_0" (1.01 ± 0.11) and the lowest "A_1"(-0.238 ± 0.03) were recorded for cows that calved in 1987 (Table 3).

The overall mean actual lactation yield, and lactation milk yield predicted from IG (b = 1) and IP are 1522.3 ± 17.18, 1540.3 ± 15.91 and 1360.9 ± 110.9

kg, respectively (Table 4). The IG (b = 1) over-predicted the overall lactation yields by about 2.2 ± 2.33 % while the IP under-predicted overall lactation yield by 15.5 ± 9.61 %. The IG (b = 1) over-predicted least square mean lactation yields by 1.1 to 5.0% for all sire breeds, dam breeds, locations and parities, while the IP under-predicted least square mean lactation yield of all sire breeds except Horro and that of both dam breeds and locations. In the IP case the percent deviations of predicted lactation yields from actual lactation yields were very high for almost all sources of variation indicating that the model is not good predictor of lactation yield. Analysis of Variance of the percent deviation obtained from IG (b = 1) indicated that it was significantly (at least $p < 0.05$) different among sire breeds, dam breeds, locations and parities. Significantly highest deviation was recorded for Horro cows (5.0 ± 0.69 %) compared to the other sire breeds. As a dam breed, the percent deviation was significantly higher for Horro than Boran cows (3.4 vs 1.1%). Similarly, higher deviation of predicted yield from actual yield was recorded for Bako herd than Debre Zeit (3.1 vs 1.5 %). Among the parities, the percent deviation was highest for cows in later than earlier parities. Similarly, the percent deviation of the predicted from actual lactation milk yield estimated from the IP was significantly ($p < 0.05$) affected by sire breed only. Significantly highest deviation was observed for Boran cows (190.9 ± 50.9 %) compared to the other breeds.

Table 1. Mean square from least square analysis of variance of "a" and "c" of the IG (b = 1) fitted to different data types

Source of variation	**IG (b =1)**		**Inverse polynomial**		
	"a"	**"c"**	**"A_0"**	**"A_1"**	**"A_2"**
Sire breed	17.6 ***	0.1 ***	40.4***	3.1 ***	0.02 ***
Calving season	2.9 ***	0.01 **	0.39NS	0.02 NS	0.0001 NS
Calving year	0.6 ***	0.01 ***	2.92***	0.2 ***	0.001 ***
Parity	3.5 ***	0.004 *	0.12 NS	0.01 NS	0.0001 NS
Error mean square	0.2	0.002	0.72	0.06	0.0003
Error df	1229	1229	1229	1229	1229
R2	37.2	19.6	21.8	20.7	20.1
CV (%)	15.8	40.2	223.4	317.7	282.7

Significance level *** = $p < 0.001$, ** = $p < 0.01$, * = $p < 0.05$ NS = not significant

Table 2. Mean square from analysis of variance of percentage deviation of predicted lactation yield from actual lactation yield estimated from IG (b =1) and IP

Source of variation	**IG (b =1)**	**IP**
Sire breed	0.04***	407125.8 *
Dam breed	0.09***	20519.9 NS
Location	0.03*	450.6 NS
Parity	0.02*	-
Error mean square	0.007	127469.5
Error df	1368	1373
R2	9.5	1.2

Significance level *** = p < 0.001, ** = p < 0.01, * = p < 0.05 NS = not significant

Discussion

Sire breed, parity, calving season and year significantly ($p < 0.001$) affected the parameters “a” and “c” (Tables 1&3). Crossbred cows had significantly higher value of “a” and lower value of “c” than the indigenous breeds. This indicates that crossbred cows start their lactation at a relatively higher level of production and the rate of decline from peak production is slower compared to the indigenous breeds. The crossbreds, however, were similar in the value of the parameter “a” and “c” while the Horro had higher “a” value compared to Boran cows. Similar breed variations in the values of the parameters “a” and “c” have been reported from previous works (Yadav *et al*., 1977; Madalena *et al*., 1979; Rao and Sundaresan, 1982; Wood, 1980; Abubakar and Buvanendran, 1981; Batra, 1986; Barios *et al*., 1996). These variations reported from different studies are related to breed differences in lactation yield. Crossbred cows had higher lactation yield (Table 4) and initial and peak milk yield (Gebregziabher *et al*., 2003a) compared to the indigenous breeds. Higher yields are associated with higher values of “a”.

The value of the parameter “c” obtained in this study is higher than what is reported by Madalena *et al.* (1979) for IG ($b \neq 1$) fitted to daily milk production and Collins-Lusweti (1991) for monthly and weekly total milk yields for Holstein Friesian and Jersey cows. Higher “c” value obtained in this study reflects faster rate of decline from peak yield in these cows compared to the reports of Collins-Lusweti (1991). The significant sire breed effects are related to additive gene contributions from the better milk production of exotic breeds. Besides, non genetic factors such as calving season and year and parity contributed to the lower “a” and higher “c” values obtained in this study compared to the reports of Collins-Lusweti (1991) due to their direct or indirect effect on the animals calving weight, body condition and energy reserve for milk production, which is in agreement to other reports (Abubakar and Buvanendran, 1981; Batra, 1986; Barrios *et al*., 1996). The higher "c" value for the indigenous breeds (Horro and Boran) than the crossbred (Friesian, Jersey and Simmental) is a direct reflexion of shorter lactation length and low lactation yield obtained in this study (Table 4) and supported by previous works (Sendros *et al*., 1987; Chernet *et al*., 2000). Lactation length is negatively correlated with the parameter "c" (Rao and Sundarsun, 1981) and cows which had shorter lactation length, showed a quicker rate of decline from the peak ("c") resulting in lower persistency (Abubakar and Buvanendran, 1981) compared to cows with longer lactation length. Similarly, Yadav *et al.* (1977) reported that the rate of decline was high in Hariana and crossbreds but not in ¾ Friesian. The positive correlation of "a" value with lactation yield, peak yield and daily yield up to peak yield (Rao and Sundaresan, 1981), the lower values of "a" for cows which had shorter lactation length (Abubakar and Buvanendran, 1981) and positive phenotypic and genotypic correlations of peak milk yield with the parameter "a" (Shanks *et al*., 1981) suggest that this coefficient could be used in genetic selection for high lactation yield.

The effect of calving season on the parameter "a" and "c" was significant (Table 1). Significantly (at least $p < 0.01$) higher "a" and "c" values were obtained for cows that calved during Arfasa (short rainy season). This is related to the availability of feed and condition of the cows at calving. The amount of milk produced by an animal is derived from immediately accessible metabolites en route through the physiological system and mobilization of body reserves (Wood, 1977). Thus, cows that calved during Arfasa (short rainy season) probably had a better body condition as a result of the steaming up which has taken place indoor during Bona (dry season), hence the cows were not exposed to severe feed shortage during the dry season. Similar calving season effect on the parameter "a" and "c" were reported by Rao and Sundaresan (1982) and Batra (1986). All these reports ascribed calving season variation to the high quality fodder during the later stages of lactation. In the study sites (Bako and Debre Zeit), cows that calved during Arfasa have a chance to exploit the good grazing condition of the pastures in Gana (wet season). Besides, those cows are in an advantage that steaming up takes place during the dry season, which can protect the animal from losing weight and condition at time of calving and during postpartum periods.

Parity of the cow significantly affected both “a” and “c”. The parameter “c” showed a linear trend with parity order (Tables 3) which is consistent with different reports (Yadav *et al.,* 1977; Shanks *et al.*, 1981; Rao and Sundaresan, 1982; Barrios *et al.,* 1996). The significantly ($p < 0.05$) lowest “c” value for cows in the earlier parities is related to higher persistency values reported for these cows than cows in the later parities. Similar trend was observed in Sahiwal and Friesian cows (Wood, 1968; Rao and Sundaresan, 1979). The parity difference obtained in this study is related to the maturity of the cows, and its ability to withstand the stresses of lactation and make use of available reserve energy for milk production (Wood, 1977). First lactation cows are undergoing a maturation process which leads in an increase in milk secreting cells with the advance in lactation that counter balances the normal decline in milk yield as the lactation progresses (Stanton *et al.*, 1992) compared to mature cows. Besides, Collins-Lusweti (1989, 1991) also attributed the parity effect to the difference in the rate of depletion of body reserves, in that mature cows use their body reserves much faster in the early stages of lactation which leads to higher “c” value than in heifers. Hence, lower “c” value for first parity cows than later parities resulted in flat lactation curves for first lactation cows than cows in later lactations (Wood, 1970).

The influence of calving year on the parameters “a” and “c” were significant (Table 1&3). The highest value for the parameter “a” and “c” were observed for cows that calved in 1989. Similar year effect was reported by Madalena *et al.* (1979) and Rao and Sundaresan (1979, 1981, 1982).

The effects of sire breed and calving year on “A_0”, “A_1” and “A_2” were significant ($p < 0.001$; Table 1&3). Similar calving year and breed effects were reported by Batra (1986) and Yadav and Sharma (1985). The effects of parity and calving season were not significant in this study, which is contradictory to the report of Batra (1986) and Yadav and Sharma (1985). The reasons for the significant breed difference in the values of the parameters “A_0”, and “A_2” are related to the breed differences in the parameters “a” and “c” discussed elsewhere in this paper. The highest value of “A_0” observed show faster rate of increase to peak and lower “A_0” indicates flatter lactation curves, which reflects slow rate of increase to peak production (Batra, 1986). Therefore, the higher "A_0" and "A_2" for indigenous than crossbred cows are explained by the shorter ascending phase and lower persistency index reported in Gebregziabher *et al*. (2003a). Kumar and Bhat (1979) reported good fit of the IP for lactations that started at lower level and peak earlier than average. Gebregziabher *et al*. (2003b) reported that the IG (b = 1) followed by IP best fit to lactation data of indigenous breeds. In the IG (b = 1), the parameter "b" which is the rate of increase to peak production is set to be one meaning that highest slop of the curve to peak or fastest rate of increase to peak. Similarly, the indigenous breeds had highest value of "A_0" (rate of increase to peak production) than the crossbred cows. In both models the data showed fast rate of increase to peak, which in other words means shorter ascending phase. Thus, this finding suggests that in case where there is no IG (b = 1) in the comparison as in the case of Kumar and Bhat (1979), the IP could be the best model to fit the data.

The IG (b = 1) over predicted (range 1.1 to 5%) lactation yields of all sire and dam breeds, locations and parities as shown in Table 4. Horro, both as a sire and dam breed, had significantly ($p < 0.001$) the highest deviation of predicted lactation yield from the actual lactation milk yield. Besides, the predicted lactation yield of Debre Zeit herds was closest to the actual yield compared to the Bako herds. Similarly, the highest deviation was recorded for cows in the sixth parity. The IP under predicted lactation yield of both indigenous and crossbred cows except Horro. Sire breed showed a significant difference in the the percentage deviation of the predicted from the actual lactation yield. Among the sire breeds, the percent deviation for Boran cows was significantly ($p < 0.05$) the highest compared to the other sire breeds. Singh *et al*. (1997) reported that the IP function under-estimated the milk yield in the first month and over-estimated yield from the second to fifth month, thereafter, the curve ran close to the average observed lactation curve. Sudarwati *et al.* (1995), however, reported that the IP to be more accurate in predicting the 305 days milk yield equivalent closer to actual yields with smaller bias percentage and error using partial records up to three months. In this study, however, the trend between the actual and predicted yield across the lactation periods was not considered. The lactation yield predicted from IP is lower and the deviation from the actual milk yield is higher than that of IG (b = 1).

Conclusion

From this study it can be concluded that

- The model parameters estimated for both lactation curve functions were affected by genetic (sire breed) and non-genetic factors such as cow parity, calving year and season and location.
- Crossbred cows had higher initial milk yield (higher “a”) and lower rate of decline from the peak or persistent yield (lower “c” and “A2”) and slower rate of increase to peak indicating longer days to peak or longer ascending phase (higher “A1”) compared to indigenous breeds.
- The suitability of the two models for predicting lactation yields of indigenous and crossbred cows were evaluated. The IG ($b = 1$) was found to be a better predictor of lactation data of both indigenous and crossbred cows.

Table 3. Least square mean (± SE) "a", "c", "A_0", "A_1" and "A_2" estimated from fitting IG (b = 1) and IP functions for indigenous and crossbred cows

Source	**N**	**IG (b = 1)**		**N**	**IP**		
		"a"	**"c"**		**"A_0"**	**"A_1"**	**"A_2"**
Overall mean	1260	2.559 ± 0.01	0.103 ± 0.001	1260	0.379 ± 0.024	-0.076 ± 0.007	0.006 ± 0.001
Sire breed		***	***		***	***	***
Friesian crosses	605	2.705 ± 0.019 [a]	0.097 ± 0.002 [b]	605	0.233 ± 0.040 [c]	-0.036 ± 0.012 [c]	0.003 ± 0.001 [c]
Jersey crosses	233	2.644 ± 0.031 [a]	0.093 ± 0.003 [b]	233	0.228 ± 0.064 [c]	-0.038 ± 0.018 [c]	0.003 ± 0.001 [c]
Simmental crosses	140	2.679 ± 0.037 [a]	0.088 ± 0.004 [b]	140	0.092 ± 0.079 [c]	-0.0003 ± 0.022 [c]	0.001 ± 0.002 [c]
Boran	65	1.923 ± 0.052 [c]	0.129 ± 0.005 [a]	65	0.828 ± 0.108 [b]	-0.162 ± 0.031 [b]	0.013 ± 0.002 [b]
Horro	217	2.166 ± 0.029 [b]	0.138 ± 0.003 [a]	217	1.181 ± 0.061 [a]	-0.306 ± 0.017 [a]	0.022 ± 0.001 [a]
Season		***	**		NS	NS	NS
Gana(June - August)	264	2.451 ± 0.027 [b]	0.110 ± 0.003 [ab]	264	0.453 ± 0.058	-0.102 ± 0.017	0.009 ± 0.001
Birra (Sept.-Nov.)	289	2.332 ± 0.026 [c]	0.106 ± 0.003 [b]	289	0.531 ± 0.055	-0.107 ± 0.016	0.008 ± 0.001
Bona (Dec. – Feb.)	295	2.369 ± 0.025 [c]	0.105 ± 0.003 [b]	295	0.541 ± 0.053	-0.105 ± 0.015	0.008 ± 0.001
Arfasa (March – May)	412	2.543 ± 0.023 [a]	0.115 ± 0.002 [a]	412	0.519 ± 0.048	-0.120 ± 0.013	0.009 ± 0.001
Parity		***	*		NS	NS	NS
1	220	2.156 ± 0.031 [c]	0.102 ± 0.003 [c]	220	0.517 ± 0.064	-0.112 ± 0.018	0.009 ± 0.001
2	218	2.423 ± 0.031 [b]	0.107 ± 0.003 [bc]	218	0.503 ± 0.065	-0.102 ± 0.018	0.008 ± 0.001
3	250	2.489 ± 0.028 [ab]	0.107 ± 0.003 [bc]	250	0.556 ± 0.061	-0.121 ± 0.017	0.009 ± 0.001
4	220	2.482 ± 0.030 [ab]	0.111 ± 0.003 [ab]	220	0.503 ± 0.064	-0.103 ± 0.018	0.008 ± 0.001
5	165	2.535 ± 0.033 [a]	0.111 ± 0.003 [ab]	165	0.489 ± 0.071	-0.111 ± 0.020	0.009 ± 0.001
6	187	2.458 ± 0.032 [ab]	0.116 ± 0.003 [a]	187	0.496 ± 0.067	-0.103 ± 0.019	0.008 ± 0.001

Table 3. Continued.

Source	**N**	**IG (b = 1)**		**N**	**IP**		
		"a"	**"c"**		**"A_0"**	**"A_1"**	**"A_2"**
Calving year		***	***		***	***	***
1980	91	2.488 ± 0.046 ^{bcd}	0.106 ± 0.005 ^{bcde}	91	0.373 ± 0.09^{e}	-0.075 ± 0.02^{def}	0.007 ± 0.002^{cdef}
1981	87	2.492 ± 0.045 ^{bcd}	0.104 ± 0.005 ^{cdef}	87	0.296 ± 0.09^{e}	-0.056 ± 0.02^{ef}	0.006 ± 0.001^{cdef}
1982	83	2.427 ± 0.047 ^{bcdef}	0.111 ± 0.004 ^{abcd}	83	0.358 ± 0.09^{e}	-0.064 ± 0.03^{def}	0.006 ± 0.002^{cdef}
1983	63	2.466 ± 0.053 ^{bcde}	0.115 ± 0.005 ^{abc}	63	0.658 ± 0.11^{bcd}	-0.138 ± 0.03^{bcd}	0.009 ± 0.002^{bcde}
1984	41	2.264 ± 0.066 ^{gh}	0.114 ± 0.007 ^{abcd}	41	0.841 ± 0.13^{abc}	-0.174 ± 0.03^{abc}	0.012 ± 0.002^{abc}
1985	49	2.438 ± 0.059 ^{bcdef}	0.114 ± 0.006 ^{abcd}	49	0.498 ± 0.12^{cde}	-0.107 ± 0.03^{bcdef}	0.009 ± 0.002^{bcdef}
1986	63	2.364 ± 0.052 ^{defgh}	0.112 ± 0.005 ^{abcd}	63	0.951 ± 0.11^{ab}	-0.183 ± 0.03^{ab}	0.011 ± 0.002^{abcd}
1987	56	2.368 ± 0.055 ^{defgh}	0.113 ± 0.006 ^{abcd}	56	1.010 ± 0.11^{a}	-0.238 ± 0.03^{a}	0.017 ± 0.002^{a}
1988	50	2.398 ± 0.058 ^{bcdefg}	0.102 ± 0.006 ^{cdef}	50	0.545 ± 0.12^{cde}	-0.129 ± 0.03^{bcde}	0.009 ± 0.002^{bcde}
1989	18	2.750 ± 0.097 ^{a}	0.132 ± 0.009 ^{a}	18	0.636 ± 0.20^{bcde}	-0.233 ± 0.05^{ab}	0.019 ± 0.004^{a}
1990	90	2.521 ± 0.046 ^{bc}	0.124 ± 0.005 ^{a}	90	0.611 ± 0.09^{cde}	-0.161 ± 0.02^{abc}	0.013 ± 0.002^{ab}
1991	53	2.311 ± 0.058 ^{efgh}	0.102 ± 0.006 ^{cdef}	53	0.398 ± 0.12^{de}	-0.065 ± 0.03^{def}	0.005 ± 0.002^{def}
1992	58	2.441 ± 0.056 ^{bcdef}	0.118 ± 0.006 ^{ab}	58	0.344 ± 0.11^{e}	-0.073 ± 0.03^{def}	0.008 ± 0.002^{bcdef}
1993	61	2.387 ± 0.055 ^{cdefg}	0.113 ± 0.005 ^{abcd}	61	0.376 ± 0.11^{de}	-0.070 ± 0.03^{def}	0.006 ± 0.002^{cdef}
1994	91	2.243 ± 0.047 ^{h}	0.100 ± 0.004 ^{def}	91	0.404 ± 0.09^{de}	-0.074 ± 0.03^{def}	0.006 ± 0.002^{cdef}
1995	100	2.456 ± 0.043 ^{bcde}	0.099 ± 0.004 ^{def}	100	0.206 ± 0.09^{e}	-0.022 ± 0.03^{f}	0.003 ± 0.001^{f}
1996	66	2.397 ± 0.054 ^{cdefg}	0.0.97 ± 0.006 ^{def}	66	0.503 ± 0.11^{cde}	-0.088 ± 0.03^{cdef}	0.005 ± 0.002^{def}
1997	75	2.543 ± 0.051 ^{ab}	0.097 ± 0.005 ^{ef}	75	0.347 ± 0.11^{e}	-0.054 ± 0.03^{ef}	0.004 ± 0.002^{ef}
1998	65	2.297 ± 0.054 ^{fgh}	0.092 ± 0.005 ^{f}	65	0.349 ± 0.112^{e}	-0.054 ± 0.03^{ef}	0.005 ± 0.002^{ef}

Means in a column within a group with different superscript vary significantly (*** = $p < 0.001$; ** = $p < 0.01$, * = $p < 0.05$ NS = not significant)

Table 4. Least square mean lactation yield (LY), lactation length (LL), predicted lactation yield (PY) and percentage deviation of predicted yield (%D) estimated from fitting the IG (b = 1) and IP to weekly total milk data

	N	LY	LL	N	IG (b = 1)		IP	
					PY	**% D**	**PY**	**% D**
Overall mean	1415	1522.3 ± 17.18	305.2 ± 2.32	1380	1540.3 ± 15.91	-2.2 ± 2.33	1360.9 ± 110.49	15.5 ± 9.61
Sire breed						***		*
Friesian crossbreds	684	1700.8 ± 29.08	318.2 ± 3.92	671	1714.9 ± 26.74	-1.7 ± 0.39 [a]	1459.9 ± 183.54	15.3 ± 15.96 [a]
Jersey crossbreds	276	1652.9 ± 46.55	313.2 ± 6.28	276	1655.0 ± 42.89	-1.6 ± 0.62 [a]	1692.6 ± 297.38	3.9 ± 25.87 [a]
Simmental	159	1820.1 ± 58.29	321.6 ± 7.86	159	1820.2 ± 53.62	-1.1 ± 0.78 [a]	1654.4 ± 371.7	11.7 ± 32.33 [a]
Boran	75	478.2 ± 79.57	234.7 ± 10.74	53	527.7 ± 85.35	-2.0 ± 1.25 [a]	68.1 ± 586.1	190.9 ± 50.9 [b]
Horro Dam	221	707.9 ± 51.78	248.9 ± 6.98	221	713.1 ± 47.52	-5.0 ± 0.69 [b]	743.3 ± 329.5	-10.8 ± 28.66 [a]
breed						***		NS
Boran	795	1440.2 ± 31.53	299.2 ± 4.25	760	1432.6 ± 30.30	-1.1 ± 0.44 [a]	1164.3 ± 208.02	47.9 ± 18.09
Horro	620	1103.8 ± 42.66	275.4 ± 5.75	620	1139.8 ± 40.19	-3.4 ± 0.59 [b]	1083.1 ± 278.1	36.5 ± 24.19
Location						*		NS
Bako Debre	907	1321.8 ± 25.76	287.3 ± 3.47	894	1341.3 ± 25.53	-3.1 ± 0.37 [a]	1110.6 ± 176.18	41.2 ± 15.32
Zeit Parity	508	1222.2 ± 49.53	287.3 ± 6.68	486	1231.1 ± 46.82	-1.5 ± 0.68 [b]	1136.7 ± 322.73	43.2 ± 28.07
1						*		
2	256	976.8 ± 44.51	288.8 ± 6.00	252	962.9 ± 41.47	-1.1 ± 0.61 [c]		
3	251	1291.0 ± 46.96	297.3 ± 6.33	244	1280.5 ± 44.58	-1.9 ± 0.65 [bc]		
4	286	1412.4 ± 44.17	294.2 ± 5.96	282	1430.4 ± 41.55	-2.5 ± 0.61 [abc]		
5	248	1439.2 ± 46.57	295.9 ± 6.28	243	1447.5 ± 43.86	-1.8 ± 0.64 [bc]		
6	177	1341.6 ± 52.54	275.4 ± 7.09	173	1372.3 ± 49.21	-2.9 ± 0.72 [ab]		
	197	1167.9 ± 50.89	272.2 ± 6.87	186	1223.3 ± 48.86	-3.6 ± 0.71 [a]		

Means in a column within a group with different superscript vary significantly (*** = $p < 0.001$; * = $p < 0.05$)

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